# CHAPTER # 05

# REDUCTION FORMULAE, VOLUME OF SOLID OF REVOLUTION

## Formula's of Definite Integrals

(1) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

(2) $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

(3) $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(2a-x)\,dx$

(4) $\int_{-a}^a f(x)\,dx = \begin{cases} 0, & \text{if } f(x) \text{ is an odd function} \\ 2\int_0^a f(x)\,dx, & \text{if } f(x) \text{ is an even function} \end{cases}$

<u>odd function</u> → $f(x)$ is said to be an odd function if $f(-x) = -f(x)$

<u>Even function</u> → $f(x)$ is said to be an even function if $f(-x) = f(x)$

<u>Exp</u>- ① $f(x) = \sin x$

$f(-x) = \sin(-x) = -\sin x = -f(x)$

Hence $f(x) = \sin x$ is an odd function

② $f(x) = \cos x$

$f(-x) = \cos(-x) = \cos x = f(x)$

Hence $f(x) = \cos x$ is an even function.

(5) $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx, \quad a < c < b$

**Reduction formula of $\int \sin^n x\,dx$.**

$\int \sin^n x\,dx = \int \sin^{n-1} x \sin x\,dx.$

$= \sin^{n-1} x \int \sin x\,dx - \int \left(d(\sin^{n-1} x) \int \sin x\,dx\right) dx$

$$\int \sin^n x\,dx = -\sin^{n-1}x\cos x + \int (n-1)\sin^{n-2}x\cos x\cos x\,dx$$

$$= -\sin^{n-1}x\cos x + (n-1)\int \sin^{n-2}x\cos^2 x\,dx$$

$$= -\sin^{n-1}x\cos x + (n-1)\int \sin^{n-2}x\,(1-\sin^2 x)\,dx$$

$$= -\sin^{n-1}x\cos x + (n-1)\int \sin^{n-2}x\,dx - (n-1)\int \sin^n x\,dx$$

$$(\cancel{1}+n\cancel{-1})\int \sin^n x\,dx = -\sin^{n-1}x\cos x + (n-1)\int \sin^{n-2}x\,dx$$

$$\Rightarrow \int \sin^n x\,dx = -\frac{\sin^{n-1}x\cos x}{n} + \frac{n-1}{n}\int \sin^{n-2}x\,dx.$$

$$\int_0^{\pi/2} \sin^n x\,dx = \begin{cases} \dfrac{n-1}{n}\cdot\dfrac{n-3}{n-2}\cdot\dfrac{n-5}{n-4}\cdots\dfrac{2}{3}, & \text{if } n \text{ is odd.} \\ \dfrac{n-1}{n}\cdot\dfrac{n-3}{n-2}\cdot\dfrac{n-5}{n-4}\cdots\dfrac{1}{2}\times\dfrac{\pi}{2}, & \text{if } n \text{ is even.} \end{cases}$$

$$= \int_0^{\pi/2} \cos^n x\,dx$$

Similarly

$$\int \cos^n x\,dx = \frac{\cos^{n-1}x\sin x}{n} + \frac{n-1}{n}\int \cos^{n-2}x\,dx.$$

**Exp. (1) Evaluate $\int \cos^7 x\,dx$.**

**Solution:**

$$\int \cos^7 x\,dx = \frac{\cos^6 x\sin x}{7} + \frac{6}{7}\int \cos^5 x\,dx.$$

$$= \frac{\cos^6 x\sin x}{7} + \frac{6}{7}\left[\frac{\cos^4 x\sin x}{5} + \frac{4}{5}\int \cos^3 x\,dx\right]$$

$$= \frac{\cos^6 x\sin x}{7} + \frac{6\cos^4 x\sin x}{35} + \frac{24}{35}\int \cos^3 x\,dx$$

$$= \frac{\cos^6 x\sin x}{7} + \frac{6\cos^4 x\sin x}{35} + \frac{24}{35}\left[\frac{\cos^2 x\sin x}{3} + \frac{2}{3}\int \cos x\,dx\right]$$

$$= \frac{\cos^6 x\sin x}{7} + \frac{6\cos^4 x\sin x}{35} + \frac{8\cos^2 x\sin x}{35} + \frac{16}{35}\sin x \quad \text{Ans.}$$

**(2)** $\int_0^{\pi} x\sin^5 x\,dx$

**Solution**

$$I = \int_0^{\pi} x\sin^5 x\,dx$$

using formula $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$, we get

$$I = \int_0^{\pi} (\pi - x)\sin^5(\pi - x)\,dx.$$

$$= \int_0^{\pi} (\pi - x)\sin^5 x\,dx \qquad [\sin(\pi-\theta) = \sin\theta]$$

$$= \int_0^{\pi} \pi\sin^5 x\,dx - \int_0^{\pi} x\sin^5 x\,dx.$$

5/2

$$I = \pi\int_0^{\pi} \sin^5 x\, dx - I$$

$$2I = \pi \int_0^{\pi} \sin^5 x\, dx$$

$$I = \frac{\pi}{2} \int_0^{\pi} \sin^5 x\, dx.$$

Now, using formula $\int_0^{2a} f(x)\, dx = \int_0^{a} f(x)\,dx + \int_0^{a} f(2a-x)\,dx,$

we get

$$I = \frac{\pi}{2}\left[\int_0^{\pi/2} \sin^5 x\, dx + \int_0^{\pi/2} \sin^5 (\pi - x)\, dx\right]$$

$$= \frac{\pi}{2}\left[\int_0^{\pi/2} \sin^5 x\, dx + \int_0^{\pi/2} \sin^5 x\, dx\right]$$

$$= \pi \int_0^{\pi/2} \sin^5 x\, dx$$

$$= \pi\, \frac{5-1}{5} \cdot \frac{5-3}{5-2} = \pi \cdot \frac{4}{5} \cdot \frac{2}{3} = \frac{8\pi}{15} \text{ Ans.}$$

(3) $\int_{-\pi/2}^{\pi/2} \sin^7 x\, dx$

Here,

$f(x) = \sin^7 x$

$f(-x) = \sin^7(-x) = -\sin^7 x$

$= -f(x)$

Hence $f(x)$ is an odd function.

$$\therefore \int_{-\pi/2}^{\pi/2} \sin^7 x\, dx = 0$$

(4) $\int_0^{2\pi} \cos^6 x\, dx$

$$\int_0^{2\pi} \cos^6 x\, dx = \int_0^{\pi} \cos^6 x + \int_0^{\pi} \cos^6 (2\pi - x)\, dx.$$

$\left[\text{Using formula } \int_0^{2a} f(x)\, dx = \int_0^{a} f(x)\,dx + \int_0^{a} f(2a-x)\,dx\right]$

$$= \int_0^{\pi} \cos^6 x + \int_0^{\pi} \cos^6 x\, dx$$

$$= 2\int_0^{\pi} \cos^6 x\, dx.$$

$$= 2\left[\int_0^{\pi/2} \cos^6 x\, dx + \int_0^{\pi/2} \cos^6 (\pi - x)\, dx\right]$$

Now,

$$\int_0^{2\pi} \cos^6 x \, dx = 2\left[\int_0^{\pi/2} \cos^6 x\, dx + \int_0^{\pi/2} \cos^6 x \, dx\right]$$

$$= 4\int_0^{\pi/2} \cos^6 x \, dx$$

$$= 4 \cdot \frac{6-1}{6} \cdot \frac{6-3}{6-2} \cdot \frac{6-5}{6-4} \cdot \frac{\pi}{2}$$

$$= \cancel{4} \cdot \frac{5}{6} \cdot \frac{3}{\cancel{4}} \cdot \frac{1}{2} \cdot \frac{\pi}{2} = \frac{15\pi}{24} \quad \underline{\text{Ans.}}$$

**(5)** $\cancel{\int_0^{\pi/4} \cos^8 x \, dx}$ $I = \int_0^{\pi/4} \cos^8 2x \, dx$

Suppose $2x = t$, $\quad$ At $x = 0$, $t = 0$

$2dx = dt$, $\quad$ At $x = \pi/4$, $t = \pi/2$

$$\therefore I = \int_0^{\pi/2} \cos^8 t \, \frac{dt}{2} = \frac{1}{2}\int_0^{\pi/2} \cos^8 t \, dt.$$

$$= \frac{1}{2} \cdot \frac{7}{8} \cdot \frac{5}{\cancel{6}_2} \cdot \frac{\cancel{3}}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2} = \frac{35\pi}{512} \quad \underline{\text{Ans.}}$$

**(6)** $I = \int_0^1 \frac{x^8}{\sqrt{1-x^2}} \, dx.$

Suppose $x = \sin\theta$, $\quad$ At $x = 0$, $\theta = 0$

$dx = \cos\theta \, d\theta$ $\quad$ $x = 1$, $\theta = \pi/2$

$$\therefore I = \int_0^{\pi/2} \frac{\sin^8\theta}{\sqrt{1-\sin^2\theta}} \cdot \cos\theta \, d\theta$$

$$= \int_0^{\pi/2} \frac{\sin^8\theta}{\cancel{\cos\theta}} \cdot \cancel{\cos\theta} \, d\theta = \cancel{\int_0^{\pi/2} \cos^8\theta \, d\theta} = \int_0^{\pi/2} \sin^8\theta \, d\theta.$$

$$= \frac{7}{8} \cdot \frac{5}{\cancel{6}_2} \cdot \frac{\cancel{3}}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2} = \frac{35\pi}{256} \quad \underline{\text{Ans.}}$$

## Reduction formula for $\int \sin^m x \cos^n x \, dx$

$$\int \sin^m x \cos^n x \, dx = -\frac{\sin^{m-1} x \cos^{n+1} x}{m+n} + \frac{m-1}{m+n}\int \sin^{m-2} x \cos^n x \, dx$$

OR

$$\int \sin^m x \cos^n x \, dx = + \frac{\sin^{m+1} x \cos^{n-1} x}{m+n} + \frac{n-1}{m+n}\int \sin^m x \cos^{n-2} x \, dx.$$

$$\int \sin^m x \cos^n x\,dx = \int \sin^{m-1}x\,(\sin x \cos^n x)\,dx$$

$$= \sin^{m-1}x \int \sin x \cos^n x\,dx - \int\left(d(\sin^{m-1}x)\int \sin x \cos^n x\,dx\right)dx$$

$$= \sin^{m-1}x\left[-\frac{\cos^{n+1}x}{n+1}\right] + \int (m-1)\sin^{m-2}x \cos x \frac{\cos^{n+1}x}{n+1}\,dx$$

$$= -\frac{\sin^{m-1}x \cos^{n+1}x}{n+1} + \frac{m-1}{n+1}\int \sin^{m-2}x \cos^{n+2}x\,dx.$$

$$= -\frac{\sin^{m-1}x \cos^{n+1}x}{n+1} + \frac{m-1}{n+1}\int \sin^{m-2}x \cos^n x \cdot \cos^2 x\,dx$$

$$= -\frac{\sin^{m-1}x \cos^{n+1}x}{n+1} + \frac{m-1}{n+1}\int \sin^{m-2}x \cos^n x(1-\sin^2 x)\,dx$$

$$= -\frac{\sin^{m-1}x \cos^{n+1}x}{n+1} + \frac{m-1}{n+1}\int \sin^{m-2}x \cos^n x\,dx - \frac{m-1}{n+1}\int \sin^m x \cos^n x\,dx$$

$\int \sin x \cos^n x\,dx$

Suppose $\cos x = t$

$-\sin x\,dx = dt$

$-\int t^n\,dt$

$= -\frac{t^{n+1}}{n+1}$

$= -\frac{\cos^{n+1}x}{n+1}$

$$\frac{m-1}{n+1}\int \sin^{m-2}x \cos^n x\,dx,$$

$$\left(1+\frac{m-1}{n+1}\right)\int \sin^m x \cos^n x\,dx = -\frac{\sin^{m-1}x \cos^{n+1}x}{n+1} + \frac{m-1}{n+1}\int \sin^{m-2}x \cos^n x\,dx$$

$$\left(\frac{n+\cancel{1}+m-\cancel{1}}{n+1}\right)\int \sin^m x \cos^n x\,dx = -\frac{\sin^{m-1}x \cos^{n+1}x}{n+1} + \frac{m-1}{n+1}\int \sin^{m-2}x \cos^n x\,dx$$

$$\Rightarrow \int \sin^m x \cos^n x\,dx = -\frac{\sin^{m-1}x \cos^{n+1}x}{m+n} + \frac{m-1}{m+n}\int \sin^{m-2}x \cos^n x\,dx.$$ Proved

Similarly $\int \sin^m x \cos^n x\,dx = \int \sin^m x \cdot \cos x \cdot \cos^{n-1}x\,dx$

$$= \int (\sin^m x \cos x)\cos^{n-1}x\,dx$$

$$= \cos^{n-1}x \int \sin^m x \cos x\,dx - \int\left(d(\cos^{n-1}x)\int \sin^m x \cos x\,dx\right)dx$$

$$= \frac{\cos^{n-1}x \sin^{m+1}x}{m+1} + \int (n-1)\cos^{n-2}x \sin x \cdot \frac{\sin^{m+1}x}{m+1}\,dx$$

$$= \frac{\cos^{n-1}x \sin^{m+1}x}{m+1} + \frac{n-1}{m+1}\int \cos^{n-2}x \sin^{m+2}x\,dx$$

$\int \sin^m x \cos x\,dx$

Suppose $\sin x = t$

$\cos x\,dx = dt$

$\int t^m dt = \frac{t^{m+1}}{m+1}$

$= \frac{\sin^{m+1}}{m+1}$

$$\Rightarrow \int \sin^m x \cos^n x\,dx = \frac{\cos^{n-1}x \sin^{m+1}x}{m+n} + \frac{n-1}{m+n}\int \sin^m x \cos^{n-2}x\,dx$$

Proved

$$\int_0^{\pi/2} \sin^m x \cos^n x\,dx = \frac{(m-1)(m-3)\cdots(n-1)(n-3)}{(m+n)(m+n-2)} \times K$$

where $K = \begin{cases} \pi/2, & \text{if } m \,\&\, n \text{ are even} \\ 1, & \text{otherwise.} \end{cases}$

**Exp** Evaluate $\int_0^{\pi/6} \sin^4 6x \cos^6 3x \, dx$

**Solution:**

Suppose $3x = t$ , [redacted]

$3\,dx = dt$ At $x = 0, \; t = 0$

$dx = dt/3$ $\quad x = \pi/6, \; t = \pi/2$

$$\therefore \int_0^{\pi/6} \sin^4 6x \cos^6 3x \, dx = \int_0^{\pi/2} \sin^4 2t \cos^6 t \, dt/3$$

$$= \frac{1}{3}\int_0^{\pi/2} \sin^4 2t \cos^6 t \, dt$$

$$= \frac{1}{3}\int_0^{\pi/2} (2\sin t \cos t)^4 \cos^6 t \, dt$$

$$= \frac{2^4}{3}\int_0^{\pi/2} \sin^4 t \cos^4 t \cos^6 t \, dt$$

$$= \frac{16}{3}\int_0^{\pi/2} \sin^4 t \cos^{10} t \, dt$$

$$= \frac{\cancel{16}}{3}\left[\frac{\cancel{3}\cdot 1\cdot 9\cdot \cancel{7}\cdot \cancel{5}\cdot \cancel{3}\cdot 1}{\cancel{14}\cdot \cancel{12}\cdot \cancel{10}\cdot \cancel{8}\cdot \cancel{6}\cdot 4\cdot 2}\cdot\frac{\pi}{2}\right]$$

$\quad\quad 2 \;\; 4 \;\; 2 \;\; 7$

$$= \frac{3\pi}{256} \text{ Ans.}$$

**Exp.** Evaluate $\int_0^{\infty} \frac{x^8 - x^5}{(1+x^3)^5} \, dx$.

**Solution:**

Suppose $I = \int_0^{\infty} \frac{x^8 - x^5}{(1+x^3)^5} \, dx$

Suppose $x^3 = \tan^2\theta$. $\quad\quad x = 0, \; \theta = 0$

$3x^2 \, dx = 2\tan\theta \cdot \sec^2\theta \, d\theta$. $\quad x = \infty, \; \theta = \frac{\pi}{2}$

$$dx = \frac{2\tan\theta \cdot \sec^2\theta \, d\theta}{3x^2}$$

$$\therefore I = \int_0^{\pi/2} \frac{\cancel{x^2}(x^6 - x^3)}{(1+\tan^2\theta)^5} \cdot \frac{2\tan\theta \cdot \sec^2\theta \, d\theta}{3\cancel{x^2}}$$

$$= \frac{2}{3}\int_0^{\pi/2} \frac{\tan^4\theta - \tan^2\theta}{\sec^{10}\theta} \tan\theta \sec^2\theta \, d\theta.$$

$$= \frac{2}{3}\int_0^{\pi/2} \frac{\tan^4\theta - \tan^2\theta}{\sec^8\theta} \cdot \tan\theta \, d\theta = \frac{2}{3}\int_0^{\pi/2} \frac{\tan^5\theta - \tan^3\theta}{\sec^8\theta} \, d\theta$$

$$I = \frac{2}{3}\left[\int_0^{\pi/2} \frac{\tan^5\theta}{\sec^8\theta}\,d\theta - \int_0^{\pi/2} \frac{\tan^3\theta}{\sec^8\theta}\,d\theta\right]$$

$$= \frac{2}{3}\left[\int_0^{\pi/2} \frac{\sin^5\theta}{\cos^5\theta} \times \cos^8\theta\,d\theta - \int_0^{\pi/2} \frac{\sin^3\theta}{\cos^3\theta} \cdot \cos^8\theta\,d\theta\right]$$

$$= \frac{2}{3}\left[\int_0^{\pi/2} \sin^5\theta\cos^3\theta\,d\theta - \int_0^{\pi/2} \sin^3\theta\cos^5\theta\,d\theta\right]$$

$$= \frac{2}{3}\left[\cancel{\frac{4.2.2.1}{8.6.4.2}} - \cancel{\frac{2.4.2}{8.6.4.2}}\right]$$

$$= 0 \quad \underline{\underline{\text{Ans.}}}$$

## Reduction formula for $\int \tan^n x\,dx$, $\int \cot^n x\,dx$

$$\int \tan^n x\,dx = \int \tan^{n-2}x \tan^2 x\,dx$$

$$= \int \tan^{n-2}x\,(\sec^2 x - 1)\,dx$$

$$= \int \tan^{n-2}x \sec^2 x\,dx - \int \tan^{n-2}x\,dx$$

$$\boxed{\int \tan^n x\,dx = \frac{\tan^{n-1}x}{n-1} - \int \tan^{n-2}x\,dx}$$

$\int \tan^{n-2}x \sec^2 x\,dx$

Suppose $\tan x = t$

$\sec^2 x\,dx = dt$

$\int t^{n-2}\,dt = \frac{t^{n-1}}{n-1}$

$= \frac{\tan^{n-1}x}{n-1}$

Similarly

$$\int \cot^n x\,dx = -\frac{\cot^{n-1}x}{n-1} - \int \cot^{n-2}x\,dx$$

$$\int_0^{\pi/4} \tan^n x\,dx = \frac{1}{n-1}\left[\tan^{n-1}x\right]_0^{\pi/4} - \int_0^{\pi/4} \tan^{n-2}x\,dx$$

$$\boxed{\int_0^{\pi/4} \tan^n x\,dx = \frac{1}{n-1} - \int_0^{\pi/4} \tan^{n-2}x\,dx}$$

$$\int_{\pi/4}^{\pi/2} \cot^n x\,dx = -\frac{1}{n-1}\left[\cot^{n-1}x\right]_{\pi/4}^{\pi/2} - \int_{\pi/4}^{\pi/2} \cot^{n-2}x\,dx$$

$$\boxed{\int_{\pi/4}^{\pi/2} \cot^n x\,dx = +\frac{1}{n-1} - \int_{\pi/4}^{\pi/2} \cot^{n-2}x\,dx}$$

**Exp:** Evaluate $\int_{\pi/4}^{\pi/2} \cot^4 x \, dx$

**Solution:** We know that

$$\int_{\pi/4}^{\pi/2} \cot^n x \, dx = +\frac{1}{n-1} - \int_{\pi/4}^{\pi/2} \cot^{n-2} x \, dx$$

$$\therefore \int_{\pi/4}^{\pi/2} \cot^4 x \, dx = +\frac{1}{3} - \int_{\pi/4}^{\pi/2} \cot^2 x \, dx.$$

$$= +\frac{1}{3} - \left[ +\frac{1}{1} - \int_{\pi/4}^{\pi/2} \cot^0 x \, dx \right]$$

$$= +\frac{1}{3} - 1 + \int_{\pi/4}^{\pi/2} dx$$

$$= -\frac{2}{3} + \left[ x \right]_{\pi/4}^{\pi/2}$$

$$= -\frac{2}{3} + \frac{\pi}{2} - \frac{\pi}{4}$$

$$= -\frac{2}{3} + \frac{\pi}{4}$$

$$= \frac{3\pi - 8}{12} \text{ Ans.}$$

## VOLUME OF SOLID OF REVOLUTION

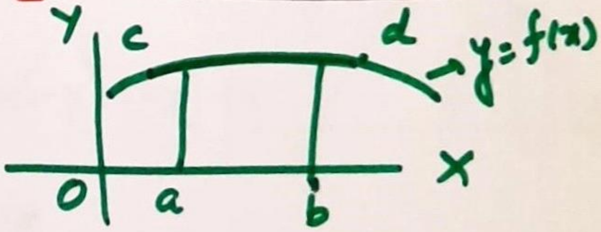
1. Volume of solid of revolution by circular disc.

If $y = f(x)$

$$V = \pi \int_a^b y^2 \, dx$$

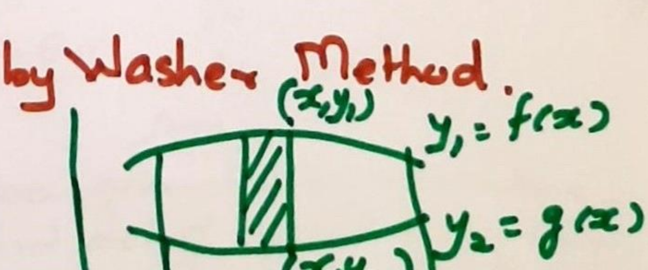
and if $x = f(y)$

$$V = \pi \int_c^d x^2 \, dy.$$



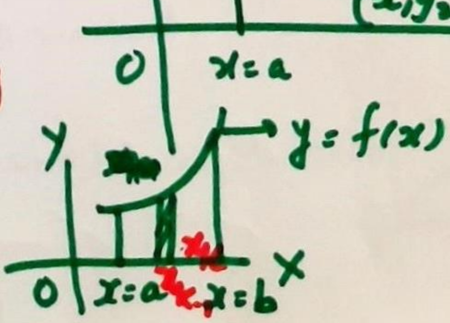

2. Volume of solid of revolution by Washer Method.

$$V = \pi \int_a^b (y_1^2 - y_2^2) \, dx$$



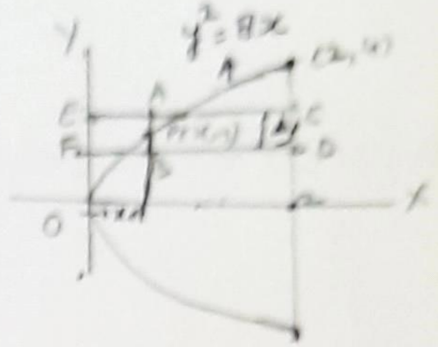

3. Volume of solid of revolution by cylindrical shell method.

$$V = 2\pi \int_a^b xy \, dx.$$



**Exp:** Find the volume generated by revolving the area bounded by the parabola $y^2 = 8x$ and it's latus rectum $(x = 2)$ about y-axis.

**Solution** Divide the area by horizontal slicing




When we revolve the area ~~at~~ about y-axis it generates a washer, whose ~~area~~ volume is generated.

= Volume of ECDF − Volume of EABF

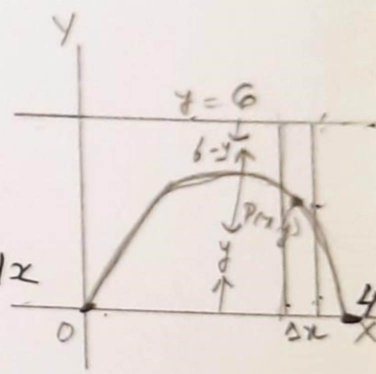
$$V = \pi\int_{-4}^{4} 4\,dy - \pi\int_{-4}^{4} x^2\,dy = \pi\int_{-4}^{4}(4 - x^2)\,dy$$

$$= \pi\int_{-4}^{4}\left(4 - \frac{y^4}{64}\right)dy = \pi\left[4y - \frac{y^5}{64\cdot 5}\right]_{-4}^{4} = \pi\left[4y - \frac{y^5}{320}\right]_{-4}^{4}$$

$$= \frac{128}{5}\pi \text{ cubic units}$$ Ans.

**Exp** Find the volume generated by revolving the area cut off from parabola $y = 4x - x^2$ by the x-axis about the line $y = 6$.

**Solution** Divide the area by vertical slicing




volume of solid of revolution by revolving given area about line $y = 6$ is a washer. The required volume will be given by

$$V = \pi\int_0^4 \left[6^2 - (6-y)^2\right]dx = \pi\int_0^4 \left[\cancel{36} - \cancel{36} - y^2 + 12y\right]dx$$

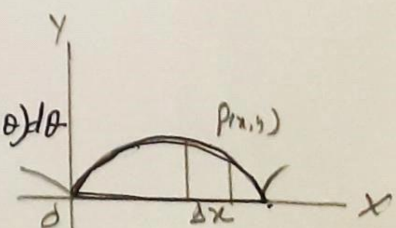
$$= \pi\int_0^4 (12y - y^2)\,dx$$

$$= \pi\int_0^4 \left[12(4x - x^2) - (4x - x^2)^2\right]dx = \pi\int_0^4 \left[48x - 12x^2 - (16x^2 + x^4 - 8x^3)\right]dx$$

$$= \pi\int_0^4 (48x - 28x^2 + 8x^3 - x^4)\,dx = \frac{1408\pi}{15} \text{ cubic units}$$

Ans.

**Exp.** Find the volume of the solid ~~of revolution~~ generated by revolving about y-axis, the area between the first arch of cycloid $x = \theta - \sin\theta$, $y = 1 - \cos\theta$ and x-axis. Use shell method.




**Solution:** $$V = 2\pi\int_0^{2\pi} xy\,dx = 2\pi\int_0^{2\pi}(\theta - \sin\theta)(1 - \cos\theta)(1 - \cos\theta)\,d\theta$$

$$V = 2\pi \int_0^{2\pi} (\theta - \sin\theta)(1 - \cos\theta)^2 d\theta .$$

$$= 2\pi \int_0^{2\pi} (\theta - \sin\theta)(1 + \cos^2\theta - 2\cos\theta)\, d\theta .$$

$$= 2\pi \int_0^{2\pi} \left[\theta + \theta\cos^2\theta - 2\theta\cos\theta - \sin\theta - \sin\theta\cos^2\theta + 2\sin^2\theta\cos\theta\right] d\theta$$

$$= 2\pi \left[\frac{\theta^2}{2} + \right.$$

$= 6\pi^3$ Cubic units. <u>Ans.</u>